Dated 16/4/20 TDC Part II Hons 8 S4b Modern Physics Paper IV th.

Radioactive Disintegration

Radioactivity is a nuclear phenomenon. The phenomenon of spontaneous emission of highly penetrating radiations from heavy elements is called radioactivity. The elements which exhibit this property are called radioactive elements. Thus radioactivity is defined as spontaneous disintegration of nuclei of certain elements with the emission of α, β and γ-rays.

Radio activity is unaffected by any external agent like temperature, high pressure, large electric and magnetic fields etc.

Laws of Radioactive Disintegration →

Rutherford and Soddy found that

1. the rate at which a particular radioactive material disintegrates was independent of physical and chemical conditions
2. The number of atoms that breaks up at any instant is proportional to the number of atoms present at that instant.

Explanation →

let N = the number of atoms present in a particular radio~~active~~ element at a given time t.

dN = number of particle disintegrate at time dt

$\frac{dN}{dt}$ = Rate of disintegration.

According to law the rate of radioactive distintegration (decay)

$-\frac{dN}{dt}$ is proportional to N

So $-\frac{dN}{dt} \propto N$ (-ve sign shows disingration)

$-\frac{dN}{dt} = \lambda N$, λ = const called disintegration cast or decay constant

$$\boxed{\lambda = \frac{-dN/dt}{N}}$$

The decay cast is a ratio of the amount of the substance which disintegrate in a unit time to the amount of the substance present.

No

$$\frac{dN}{dt} = -\lambda N$$

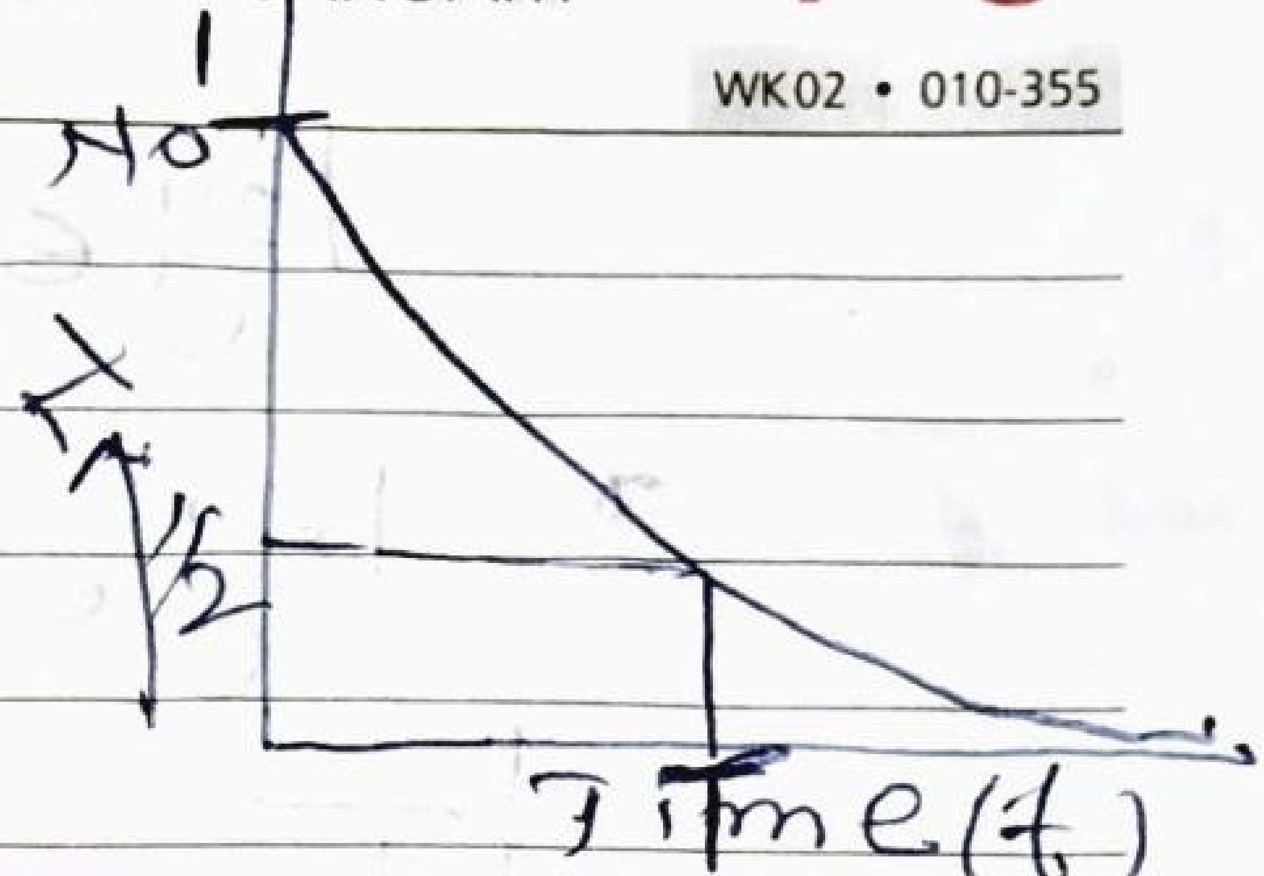
so $\frac{dN}{N} = -\lambda dt$

$\log_e N = -\lambda t + C$ (C) Const of integration

when $t = 0$ $N = N_0$

$\log_e N_0 = C$

So $\log_e \frac{N}{N_0} = -\lambda t$



$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\boxed{N = N_0 e^{-\lambda t}}$$

This is equation of Radioactive disintegra
This eqn shows that the number of atoms of a given radioactive substance decreases exponentially with time.

Half life time (T) → The time required for one-half of the radioactive subtance disintegrate is called half life time or The half life time a radioactive element is defined as the time period during which a given number of radio-active atoms is reduced to half.

~~we~~

$$N = N_0/2$$

~~No~~ at $N = N_0 e^{-\lambda t}$

$t = T/2$ $N = N_0/2$

$$\therefore \frac{\cancel{N_0}}{2} = \cancel{N_0} e^{-\lambda T/2}$$

$$\frac{1}{2} = e^{-\lambda T/2}$$

$$\log_e \tfrac{1}{2} = -\lambda T_{1/2}$$

or $$\log_e 2 = \lambda T_{1/2}$$

$$\boxed{T_{1/2} = \frac{\log_e 2}{\lambda}} = \frac{2.302 \log_{10} 2}{\lambda}$$

$$\boxed{T_{1/2} = \frac{0.693}{\lambda}} \qquad \log_{10} 2 = 0.301$$

($T_{1/2}$) Half life time is inversely proportional to decay constant (λ).

$T_{1/2}$ & λ depend upon the radioactive material. Both are completely independent of the external factors, like temp, pressure, chemical combinations etc.

Mean Average life ($\bar{t}$) of a radio-active atom. →

The Mean-life ~~time~~ of a radio ~~atom~~-active element is the ratio of the total time of all the radioactive atoms to the total number of such atoms in it.

$$\text{The Mean life of a radioactive element } (\bar{t}) = \frac{\text{Sum of the lives of all the atoms}}{\text{Total number of atoms}}$$

let N = number of atoms which have taken t seconds.

dN = number of atoms which have decaying in the next small time interval dt

Total life time of all the N_0 atoms

$$\int_{N_0}^{0} t\,dN$$

Now Mean life

$$\bar{t} = \frac{\text{Total life-time}}{\text{Total number of atoms}}$$

$$= \frac{\int_{N_0}^{\infty} t\,dN}{N_0}$$

or $N = N_0 e^{-\lambda t}$

$$dN = -N_0 \lambda e^{-\lambda t} = -\lambda N_0 e^{-\lambda t}$$

So

$$\bar{t} = = \frac{\int_0^{\infty} -\lambda N_0 e^{-\lambda t} t\,dt}{N_0}$$

$$= \frac{-\lambda \cancel{N_0}}{\cancel{N_0}} \int e^{-\lambda t} t\,dt$$

$$\bar{t} = -\lambda \left[\frac{t e^{-\lambda t}}{\lambda} - \int \frac{e^{-\lambda t}}{-\lambda} dt \right]_0^{\infty}$$

$$= - \lambda \left[\frac{e^{-\lambda t} t}{\lambda} - \frac{e^{-\lambda t}}{\lambda^2} \right]_0^{\infty}$$

$$(\bar{t}) = -\cancel{\lambda} \left(\frac{1}{\lambda \cancel{t}} \right) = -\frac{1}{\lambda}$$

Thus the Mean life time ($\bar{t}$) of a radioactive substance is the reciprocal of the decay constant (λ). (+ve) sign shows decaying of the number of atoms with time.

Relation between Half life time (T) and the Mean life ($\bar{t}$)

$$\text{Half life time } T = \frac{0.693}{\lambda}$$

$$\bar{t} = 1/\lambda$$

$$\text{So} \quad T = 0.693\,\bar{t}$$

$$\bar{t} \cancel{= 0.693/T = 1.44}$$

$$\bar{t} = T/0.693 = 1.44T$$

$$\therefore \boxed{\bar{t} > T}$$